When reporting demographic disparities, commentators ignore a basic statistical relationship and draw misleading conclusions.

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Divining Difference

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There are few statistical phenomena that are at once so fundamental and so widely misunderstood as the seemingly paradoxical relationship between disparities in the rates at which two groups succeed at something and disparities in the rates at which the same two groups fail at something. The misunderstanding of that relationship is responsible for immense confusion in the appraisal of a wide range of phenomena disparately affecting different demographic groups.

The relationship can be stated as follows: When two groups differ in their susceptibility to some condition, the less prevalent the condition, the greater will be the disparity in the rates at which the two groups are affected by the condition and the smaller will be the disparity in the rates at which the groups avoid the condition.

A Simple Example

The relationship can be easily illustrated with textbook data on test scores. Imagine two groups with normal distributions of test scores, with Group A having an average test score of approximately one-half a standard deviation higher than Group B, and the two groups having the same standard deviation. This means that roughly 30% of Group B will score above the average for Group A. If we set the cutoff at a point where 50% of Group A fails the test (point X in Fig. 1), then 70% of Group B will fail the test. Thus, Group B's failure rate is 1.4 times Group A's failure rate (70 over 50), and Group B's pass rate is 60% of Group A's pass rate (30 over 50).

If we now lower the cutoff to a point where only 30% of Group A fails the test (point Y in Fig. 1), 50% of Group B would fail the test. At this point, then, Group B's failure rate is 1.7 times Group A's failure rate (50 over 30), whereas Group B's pass rate is 71% of Group A's pass rate (50 over 70). So the gap between failure rates has increased, but the gap between pass rates has declined.

Viewed another way, as a result of the lowering of the cutoff, Group A experiences a higher proportionate decline in its failure rate (40%; i.e., 50 reduced to 30) than Group B (29%; i.e., 70 reduced to 50), but Group B experiences a higher proportionate increase in its pass rate (67%; i.e., 30 increased to 50) than Group A (40%; i.e., 50 increased to 70).

The changing relationships just described are summarized in Table 1. Corollaries to these relationships reflected in the proportion that Group B makes up of persons

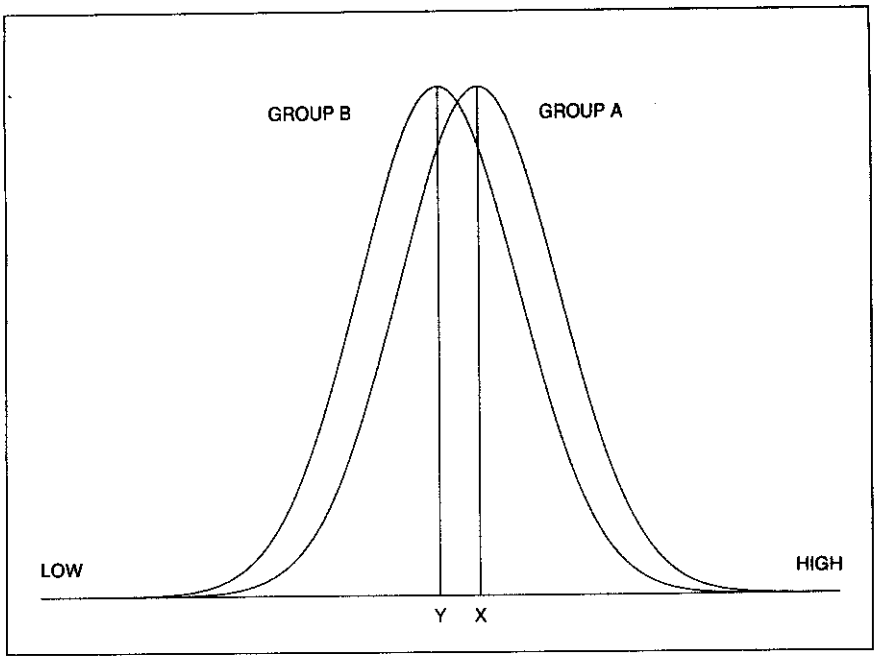

Figure 1. Normal distributions of test scores with one-half standard deviation difference in mean scores.

Table 1—Relationships Between Two Groups' Fail Rates and Pass Rates at Various Cutoff Points

	Fail Rates			Pass Rates			Percentage Group B Represents of Total Who	
Cut Score	Group B	Group A	Ratio B/A	Group B	Group A	Ratio B/A	Fail	Pass
X	70	50	1.4	30	50	0.60	58	38
Y	50	30	1.7	50	70	0.71	63	41

who fail and who pass at each cutoff point (assuming that the groups are of equal size) are shown in the last two columns of Table 1. Note that lowering the cutoff causes Group B to make up a larger proportion of the persons who fail the test but also a larger proportion of persons who pass the test.

Because lowering cutoffs reduces disparities in pass rates, lowering cutoffs is universally regarded as reducing the differential impact of a test on which one group does not perform as well as another, even though lowering cutoffs also increases disparities in failure rates. It should be recognized as well, however, that exactly the same changes would occur if, instead of lowering the cutoff, education is improved sufficiently so that everyone scoring between the two cutoff points is enabled to score above the higher cutoff.

Just the opposite changes would occur, however, if cutoffs were raised or education worsened. That is, disparities in failure rates would decline, although disparities in success rates would increase. Group A would also experience a larger increase in failure rates, but Group B would experience a larger decline in pass rates.

Thus, if one wishes to evaluate how improvements or declines in education may differentially affect two groups, reference to dichotomous variables can lead to opposite conclusions, depending on whether one focuses on success or failure. As a rule, reference to other measures such as changes in mean scores will be more enlightening.

Some Common Examples

Few real-life situations offer the precise predictability of hypothetical test data. Yet the described tendencies apply whenever two groups have different but more or less normal distributions of factors associated with the susceptibility to some condition.

For one example, each of these tendencies can be readily observed in published income data, even though income patterns depart significantly from the normal distribution. For example, in 1990, if the amount of poverty in the population had declined sufficiently to allow everyone with an income above 50% of the poverty line to be raised from poverty, the ratio of the black poverty rate to the white poverty rate would have risen from 3.0 (31.9/10.7) to 3.8 (14.4/3.8). But the black rate of avoiding poverty would have risen from 76% of the white rate to 89% of the white rate.

Despite this fact, in the late 1970s, following the period from 1959 through the middle 1970s when the United States experienced dramatic and consistent declines in poverty, much was made of the fact that disparities between the poverty rates of more- and less-advantaged groups were increasing. Rarely was there recognition of the extent to which such

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changes were the nearly inevitable consequences of declining poverty. With poverty on the rise in more recent years, it has been increasingly reported that poverty is growing faster among more-advantaged groups. It goes unrecognized that when poverty increases, it almost always increases at greater rates among groups least prone to poverty.

Once again, appraising changes in the relative economic status of two groups by reference to a dichotomous variable would likely result in different conclusions, depending on whether one focused on falling into poverty or avoiding poverty. Reference to changes in continuous measures such as mean income levels would likely be more useful. At any rate, however, it is essential that one recognize that increasing (or decreasing) disparities among the poverty rates of different demographic groups do not necessarily reflect a meaningful change in relative economic well-being.

Just as an improving economy causes poverty to be increasingly restricted to the most-disadvantaged elements of the population, advances in medicine and health care cause avoidable mortality to be increasingly restricted to those most susceptible. Given the correlation between income and health status, this tends to cause avoidable mortality to be increasingly restricted to disadvantaged segments of the population.

For about a decade, there has been recurring attention to the fact that although infant mortality has been declining generally, the racial disparity in infant mortality has been increasing. No one appeared to understand, however, that the coincidence of these phenomena was close to inevitable. Nor did anyone observe that while racial disparities in infant mortality rates were reaching all-time highs, racial disparities in infant survival rates were reaching all-time lows.

These results apply quite broadly. Just as test scores and income—factors associated with the dichotomous outcomes of test failure/passage or poverty/poverty avoidance—tend to be approximately normally distributed, factors associated with the likelihood of being affected by or avoiding any condition may also be more or less normally distributed. This suggests that when we endeavor to measure the differential effect of various therapies on groups that are more and less prone to some disease—as has been recently mandated for research funded by the National Institutes of Health—we should expect to see greater reductions in mortality among less-susceptible groups and greater increases in survival rates among more-susceptible groups. Similarly, when we interpret trends, such as the much-discussed greater rates of increases in AIDS cases among the least-susceptible groups such as women and teenagers, we should be mindful that when a condition increases, it tends usually to increase at greater rates among the least-susceptible groups.

Conclusions

This discussion is not intended to question the utility of studying demographic differences, especially in the appraisal of programs aimed at addressing those differences, but merely to emphasize that the measurement of differences is more difficult than is often realized. The tendencies just described imply that, in appraising changes in the relative status of two groups, measures other than dichotomous variables might be more useful, as suggested in the examples on test scores and poverty. When resort to dichotomous variables is unavoidable, such an appraisal must be undertaken with a full understanding of the extent to which certain changes are the inevitable or nearly inevitable consequence of the overall changes in the prevalence of a condition.